

Electrostatic Oscillations Along Cylindrical Micelles

Afshin Moradi

Received: 6 March 2011 / Accepted: 20 June 2011 / Published online: 8 July 2011
© Springer Science+Business Media, LLC 2011

Abstract Theoretical analysis of ionic electrostatic oscillations of a charged cylindrical micelle is presented within the framework of the fluid theory for surface counterions of the cell, in conjunction with the Poisson equation. General expression of dispersion relation is obtained for electrostatic excitations of system.

Keywords Membrane electrostatics · Biophysics · Measurement · Electrical aspects/membrane structure

Membranes appear in flat, spherical, tubular or tortuous, and possibly bicontinuous forms in the cell, depending on function and composition. The surface of a cell membrane in aqueous environment becomes negatively charged (Hille 2001). Thus, a thin two-dimensional (2D) layer of mobile cations is accumulated adjacent to the extra/intracellular membrane surfaces (Gouy 1910; Chapman 1913; Debye and Hückel 1923a, b; Stern 1924; Grahame 1947; Teissié et al. 1985; Prats et al. 1987; Frohlich 1986; Fisun 1993; Moradi 2011).

More recently, we calculated the eigenfrequencies of ionic electrostatic excitations of charged planar and spherical biological membranes in the hydrodynamic

approximation, taking into account the strong Coulomb interaction in the assembly of two thin layers of ions separated by a lipid membrane (Moradi 2011).

In the present work, we consider a cylindrical micelle (Israelachvili 1992). Our purpose is to develop the approximate theoretical model, to study the collective charge response for surface counterions of the cylindrical micelles. We assume that we are in the weak electrolytes regime, so the counterion charges are distributed according to the Gouy–Chapman model. In the Gouy–Chapman theory (in thermal equilibrium), the counterion charge density will be a function of counterion distance from membrane surface, within in the so-called Gouy–Chapman length

$$\lambda_{GC} = \frac{2\varepsilon_0\varepsilon_w k_B T}{\sigma q},$$

where K_B is the Boltzman constant and T is the ion temperature. Here ε_w is the dielectric constant of the outside of the cell, in absence of added salt. Also, q and $\sigma = qn_0$ are the counterion charge and the surface density, respectively.

The model system is a cylindrical cell membrane (as shown in Fig. 1) with a radius a and dielectric constant $\varepsilon_{oil} = 2$ coming essentially from the closely packed hydrocarbon (oily) tails. We consider that the surface of the system to consist of surface counterion fluid superimposed at $r = a$ with charge $q = Ze$ (Z is the ionic valence and e is the electron charge). Let us assume that the equilibrium electronic density of counterion charge fluid over the surface of the system (per unit area) be n_0 . The external region of system ($r > a$) is characterized by dielectric constant $\varepsilon_w = \varepsilon_{water} = 80$. We use cylindrical coordinates $\mathbf{x} = (r, \phi, z)$ and consider plasma oscillations with frequency ω along the cylindrical cell axis z . The homogeneous counterion fluid will be perturbed by the plasma wave and can be regarded as a charged fluid with the velocity field $\mathbf{u}(\mathbf{x}, t)$ and the

A. Moradi (✉)
Department of Nano Science, Kermanshah University
of Technology, Kermanshah, Iran
e-mail: a.moradi@kut.ac.ir

A. Moradi
Department of Nano Science, Institute for Studies in Theoretical
Physics and Mathematics (IPM), Tehran, Iran
e-mail: a.moradi@nano.ipm.ac.ir

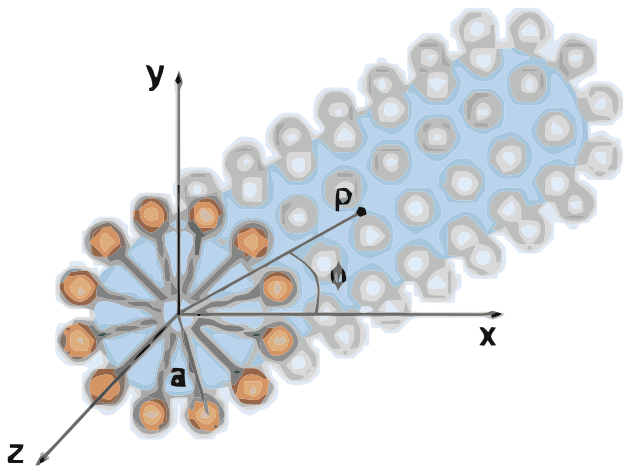


Fig. 1 Cross-sectional view of a cylindrical micelle. The dielectric constants ϵ_{oil} and ϵ_w are for the membrane and external region, respectively

perturbed density (per unit area) $n(\mathbf{x}, t)$, where $\mathbf{x} = (\phi, z)$ are the coordinates of a point at the cylindrical surface of the system. Note that the velocity field $\mathbf{u}(\mathbf{x}, t)$ has only tangential components to the cell surface. Based on the linearized hydrodynamic model (Moradi 2011), the counterion excitations on the cylindrical surface can be described by the continuity equation

$$\frac{\partial n(\mathbf{x}, t)}{\partial t} + n_0 \nabla \cdot \mathbf{u}(\mathbf{x}, t) = 0, \tag{1}$$

and the equations of the linearized momentum

$$\frac{\partial \mathbf{u}(\mathbf{x}, t)}{\partial t} = -\frac{Ze}{m_c} \nabla \Phi(\mathbf{x}, t) |_{r=a} - \frac{s^2}{n_0} \nabla n(\mathbf{x}, t) - \gamma \mathbf{u}(\mathbf{x}, t). \tag{2}$$

Also, by considering the Poisson equation, we have

$$\nabla^2 \Phi(\mathbf{x}, t) = 0, \quad r \neq a \tag{3}$$

where Φ is the self-consistent electric potential and m_c is the mass of the counterion fluid. In the right-hand side of Eq. 2, the first term is the force on counterion fluid due to the tangential component of the electric field, evaluated on the cylindrical surface $r = a$. The second term is the force due to the local compression of the counterion 2D gas, and the last term represents the collision effects of the carriers, with γ being the friction coefficient. $s = \sqrt{2k_B T/m_c}$ is the velocity of density fluctuations in the charged subsystems.

To solve the system of Eqs. 1–3, we have to provide appropriate boundary conditions. We denote Φ_1 and Φ_2 , the potential in the region $r < a$, and $r > a$. These boundary conditions are given by

$$\Phi_1(\mathbf{x}) |_{r=a} = \Phi_2(\mathbf{x}) |_{r=a} \tag{4}$$

and

$$\epsilon_w \frac{\partial \Phi_2(\mathbf{x})}{\partial r} |_{r=a} - \epsilon_{oil} \frac{\partial \Phi_1(\mathbf{x})}{\partial r} |_{r=a} = -\frac{Ze}{\epsilon_0} n. \tag{5}$$

Assuming a harmonic dependence for all quantities as a function of time, i.e. $\exp(-i\omega t)$. By eliminating the velocity $\mathbf{u}(\mathbf{x}, t)$, one can obtain the following equation from Eqs. 1 and 2:

$$\left(\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} \right) n(\mathbf{x}, t) = s^2 \nabla^2 n(\mathbf{x}, t) + \frac{Zen_0}{m_c} \nabla^2 \Phi(\mathbf{x}, t). \tag{6}$$

Due to the cylindrical symmetry of system, one can replace the quantities $n(\mathbf{x})$ and $\Phi(\mathbf{x})$ in Eqs. 6 and 3 by expressions of the form

$$n(\phi, z) = \tilde{n} \exp(im\phi) \exp(ikz) \tag{7}$$

and

$$\Phi(r, \phi, z) = \tilde{\Phi}(r) \exp(im\phi) \exp(ikz), \tag{8}$$

where m is the integer azimuthal quantum number and k is the longitudinal wave vector. After substitution, one finds

$$\left[\omega(\omega + i\gamma) - s^2 \left(k^2 + \frac{m^2}{a^2} \right) \right] \tilde{n} = \frac{Zen_0}{m_c} \left(k^2 + \frac{m^2}{a^2} \right) \tilde{\Phi}(a) \tag{9}$$

and

$$\frac{\partial^2 \tilde{\Phi}(r)}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\Phi}(r)}{\partial r} - \left(k^2 + \frac{m^2}{a^2} \right) \tilde{\Phi}(r) = 0 \quad r \neq a \tag{10}$$

The solution of Eq. 10 in our problem has the general form

$$\begin{aligned} \tilde{\Phi}_1(r) &= A_m I_m(kr), \quad r < a \\ \tilde{\Phi}_2(r) &= B_m K_m(kr) \quad r > a \end{aligned} \tag{11}$$

Substituting Eq. 11 into boundary conditions (Eqs. 4 and 5), by using Eq. 9, one can obtain the following dispersion equation:

$$\begin{aligned} \omega = & \left\{ s^2 \left(k^2 + \frac{m^2}{a^2} \right) + \omega_p^2 \frac{a}{k} \left(k^2 + \frac{m^2}{a^2} \right) \right. \\ & \left. \times \frac{I_m(ka)K_m(ka)}{\epsilon_{oil} I'_m(ka)K_m(ka) - \epsilon_w I_m(ka)K'_m(ka)} - \frac{\gamma^2}{4} \right\}^{1/2} - i \frac{\gamma}{2}, \end{aligned} \tag{12}$$

where

$$\omega_p^2 = \frac{Z^2 e^2 n_0}{\epsilon_0 m_c a}.$$

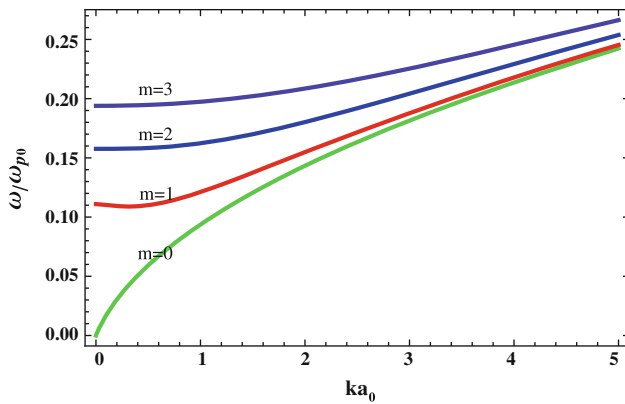


Fig. 2 The dimensionless ionic electrostatic resonances of a charged cylindrical micelle vs. dimensionless variable ka_0 , when $Z = 1$, $\sigma = qn_0 = 1 \text{ C m}^{-2}$, $s = 2.27 \times 10^3 \text{ m/s}$, $a = 4a_0$, and $\gamma = 0$

Formally speaking, the dispersion characteristics of the electrostatic oscillations along cylindrical micelles is dependent on the radius of the system, the wave number, the angular momentum, and the friction coefficient.

To clearly see behavior of the dispersion relation for the electrostatic oscillations, we plot dimensionless frequency ω/ω_{p0} versus dimensionless variable ka_0 ($a_0 = 1 \text{ nm}$) in Fig. 2, for a cylindrical micelle characterized by $a = 4a_0$ and different values of m , when $\omega_{p0}^2 = Z^2 e^2 n_0 / \epsilon_0 m_c a_0$, $Z = 1$, $\sigma = qn_0 = 1 \text{ C m}^{-2}$, $m_c = 1.67 \times 10^{-27} \text{ kg}$, $s = 2.27 \times 10^3 \text{ m/s}$, and $\gamma = 0$. One can see that the dispersion curves ω/ω_{p0} for the system continue to increase with increasing value of ka_0 for all m .

In summary, we have calculated the eigenfrequencies of ionic electrostatic excitations of a charged cylindrical micelle in the hydrodynamic approximation. The interaction between the carriers has been described in terms of

self-consistent electric field in the non retardation limit. These surface-charge oscillations may play a fundamental role in the dynamic response of cell biology to electromagnetic radiation.

References

- Chapman DL (1913) Theory of electrocapillarity. *Philos Mag* 25:475–481
- Debye P, Hückel E (1923a) The theory of electrolytes. I. *Phys Z* 24:185–206
- Debye P, Hückel E (1923b) The theory of electrolytes. II. *Phys Z* 24:305–325
- Fisun OI (1993) 2D plasmon excitation and nonthermal effects of microwaves on biological membranes. *Bioelectromagnetics* 14:57–66
- Frohlich H (1986) Coherent excitations in active biological system. In: Gutmann F, Keyser H (eds) *Modern bioelectrochemistry*. Plenum, New York, p 4
- Gouy G (1910) Sur la constitution de la charge á la surface d'un électrolyte. *J Phys* 9:457–468
- Grahame DC (1947) The electrical double layer and the theory of electrocapillarity. *Chem Rev* 41:441–501
- Hille B (2001) *Ionic channels of excitable membranes*. Sinauer Associates, Sunderland
- Israelachvili JN (1992) *Intermolecular surface forces*, 2nd edn. Academic Press, London
- Moradi A (2011) Ionic electrostatic excitations along biological membranes. *Phys Plasmas* 18:022112
- Prats M, Tocanne JF, Teissié J (1987) Lateral proton conduction at a lipid/water interface. Effect of lipid nature and ionic content of the aqueous phase. *Eur J Biochem* 162:379–385
- Stern O (1924) Zur Theorie der elektrolytischen Doppelschicht. *Z Elektrochem* 30:508–516
- Teissié J, Prats M, Soucaille P, Tocanne JF (1985) Evidence for conduction of protons along the interface between water and a polar lipid monolayer. *Proc Natl Acad Sci USA* 82:3217–3221